



K25U 0160

Reg. No. :

Name :

**Sixth Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/
Supplementary/Improvement) Examination, April 2025
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
6B12 MAT : Numerical Methods, Fourier Series and Partial Differential
Equations**

Time : 3 Hours

Max. Marks : 48

I. Answer **any 4** out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

- 1) Define first divided difference formula.
- 2) Define shift operator E.
- 3) Define explicit single step method to solve an ordinary differential equation.
- 4) Define the fundamental period.
- 5) Write one dimensional wave equation.

II. Answer **any 8** questions out of 11 questions. **Each** question carries **2** marks. **(8×2=16)**

- 6) Prove that $\Delta = E - 1$.
- 7) Prove that $\Delta \left(\frac{f_i}{g_i} \right) = \frac{g_i \Delta f_i - f_i \Delta g_i}{g_i g_{i+1} + 1}$.
- 8) Prove that $\Delta (f_i^2) = (f_i + f_{i+1}) \Delta f_i$.
- 9) Describe quadratic interpolation.
- 10) State Existence Uniqueness theorem for an initial value problem.

P.T.O.



- 11) Explain Taylor series method.
- 12) If $f(x)$ and $g(x)$ have period p then prove that $af(x) + bg(x)$ with any constant a and b also has the period p .
- 13) Define a periodic function with an example.
- 14) Solve $u_{xx} - u = 0$ like an ordinary differential equation.
- 15) State fundamental theorem on super position.
- 16) Verify that $u = \frac{y}{x}$ satisfies $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$ with $f(x, y) = \frac{2y}{x^3}$.

III. Answer **any 4** questions out of 7 questions. **Each** question carries **4** marks. **(4×4=16)**

- 17) Find the Lagrange interpolating polynomial at $x = 3$ for

$$\begin{array}{lcl} x & : & 2.5 \quad 3.5 \\ f(x) & : & 6 \quad 8 \end{array}$$

- 18) Prove that $E^{1/2} \delta = E - 1$.
- 19) Find the classical Runge-Kutta fourth order of $y' = x(y - x)$, $y(2) = 3$ in the interval $[2, 2.4]$ with step size $h = 0.2$.
- 20) Obtain the approximate value of $y(0.2)$ for the initial value problem $y' = x^2 + y^2$, $y(0) = 1$ with $h = 0.1$ using Euler method.
- 21) Find the Fourier coefficient of the periodic function $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$
and $f(x + 2\pi) = f(x)$. Hence prove that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.
- 22) Find the d'Alembert's solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$.
- 23) Find the type, transform to normal form and solve $u_{xx} + 4u_{yy} = 0$.



IV. Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

(2×6=12)

24) Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$.

25) Find the solution of the initial value problem $y' = 2y - x$, $y(0) = 1$ by performing three iteration of the Picard's method.

26) Find the even half range expansion of the function $f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$

27) Find the temperature $u(x, t)$ in a laterally insulated copper bar 80cm long if the initial temperature is $100 \sin\left(\frac{\pi x}{80}\right)^\circ \text{C}$ and the ends are kept at 0°C . How long will it take for the maximum temperature in the bar to drop to 50°C ? Physical data for copper : density 8.92g/cm^3 , specific heat $0.092 \text{cal/g}^\circ \text{C}$, thermal conductivity : $0.095 \text{cal/(cmsec}^\circ \text{C)}$.